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Pressure Response of Line Sources

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ABSTRACT

The on-axis pressure response of a vertical line source is known to decrease at 3dB per doubling of distance in the near field and at 6dB in the far field. The present paper shows that the conventional mathematics used to achieve this result understates the distance at which the -3dB to -6dB transition occurs. An examination of the pressure field of a line source reveals that the near field extends to a greater distance at positions laterally displaced from the centerline, normal to the source. The paper introduces the "endpoint" convention for the pressure response and compares the on-axis response of straight and hybrid line sources.

1. INTRODUCTION

Vertical line arrays of loudspeakers continue to grow in popularity. This is due in part because loudspeaker manufacturers have recently undertaken development of systems designed specifically to optimize the performance of line arrays. Vertical line arrays provide narrow vertical polar response, high directivity index, and smooth horizontal coverage compared to horizontal arrays of loudspeakers.

The directional performance of line arrays is well documented [1,2,3]. The literature show that the polar response narrows both with increasing length and frequency. These relationships are based on analyses of *idealized* line sources[a]. In general, the new generation of line array loudspeaker systems provides an improved approximation to a continuous line source than their predecessors.

A review of the literature reveals that the polar response of a line source is generally derived under far field conditions. This qualification simplifies the mathematics and provides closed-form solutions for a line source's directivity function and quarter-power angle. For practical purposes, the results obtained are useful. This is because far field conditions are met for many nominal-sized line arrays over a broad frequency range. However, when very long arrays are used in close proximity and/or at very high frequency, it is important to consider at what distance far field conditions are obtained.

The far field is generally accepted as the region in which the pressure response decreases at 6dB per doubling of distance. The point at which this occurs is commonly (and confusingly[b]) referred to as the *critical distance*. Up to this point, that is, between the source and the critical distance, the on-axis pressure response undulates and generally decreases at 3dB per doubling of distance.

[a] This paper uses the convention that a line *source* is an idealized, continuous acoustic radiator while a line *array* is an array of loudspeakers arranged to approximate a line source. Loudspeakers and enclosures have inherent frequency response, directional characteristics, and diffraction and shadowing effects, all which modulate the theoretical response of a pure line source.

[b] The critical distance is also defined in the context of architectural acoustics as the point at which the direct sound and the reverberant sound are at the same level. See Davis, "Sound System Engineering," Howard W. Sams and Co., Indianapolis, IN., 1975.

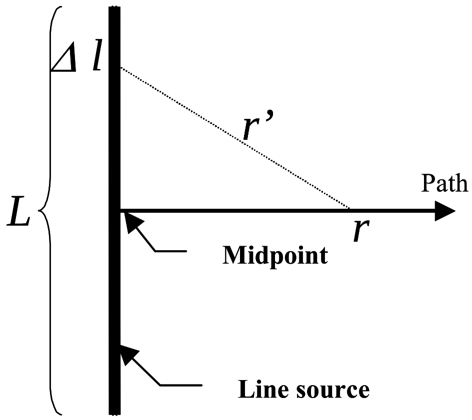

Figure 1: Geometric construction for calculating the on-axis pressure response of a line source (midpoint convention).

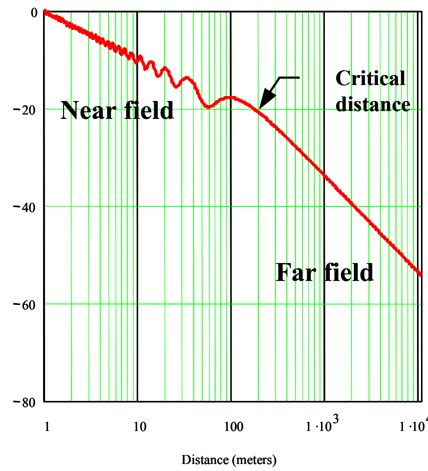

Figure 2: On-axis pressure response (from the midpoint) of a uniform line source (4m, 8kHz).

Beyond these generalizations, the literature offering a detailed analysis of a line source's on-axis pressure response is somewhat obscure. Texts generally provide the on-axis response of *piston sources* but not of *line* sources. Some estimate the distance to the far field of "linelike or pistonlike" sources [4], but do not provide the on-axis response per se. Recently, a few papers [5,6] have described with more specificity how the on-axis pressure response varies with source length and frequency. In addition, they show how the on-axis pressure response of a continuous line source is affected by gaps and other discontinuities.

A common convention among the literature is that the on-axis pressure response, whether for a piston or line source, is taken along a path normal to the source *beginning at its midpoint*. The present paper shows that this convention potentially understates the distance to the far field. Furthermore, many of the generalizations regarding the behavior of the pressure response are *unique* to the on-axis path from the midpoint.

Finally, many "line arrays" used in practice today are not "straight" line arrays at all. They are often curved slightly, generally at the lower end. This paper concludes by deriving the pressure response of several hybrid line sources and compares them to straight-line sources.

2. ON AXIS RESPONSE OF A UNIFORM LINE SOURCE - CONVENTIONAL METHOD

We will begin with a brief review of the on-axis response of a line source. It is obtained at an arbitrary distance r by summing the pressure radiated along the source. Figure 1 shows the geometric construction used to solve for the pressure along a path normal to the source, beginning at its midpoint. Referring to Figure 1, L is the total length of the source and r' is the distance from any radiating element Δl of the source to any point r along the path. Assuming uniform amplitude and phase along the line source[c], the pressure at r is given by

$$p_{mid}(r) = \int_{-L/2}^{L/2} \frac{e^{-jkr'(r,l)}}{r'(r,l)} dl$$

where

$$r'(r,l) = \sqrt{r^2 + l^2}$$

and p_{mid} refers to the pressure along a path beginning at the midpoint of the line source. The pressure *response* is the logarithmic ratio of the magnitude of the pressure squared at r over the magnitude of the pressure squared at some reference distance, i.e.

$$Pressure\ response(r) = 20\log\frac{|p(r)|}{|p(r_{ref})|}.$$

The on-axis pressure response of 4-meter line source at 8kHz is shown in Figure 2. The pressure response generally decreases at a rate of 3dB per doubling of distance out to approximately 100 meters. It exhibits undulations across this region, the magnitudes of which increase as the distance

[c] The general form of the pressure response is

$$p(r) = \int_{-L/2}^{L/2} \frac{A(l)e^{-j(kr'(l)+\phi(l))}}{r'(l)} dl$$

where $A(l)$ and $\phi(l)$ are the amplitude and phase distributions along the source. In the present paper only *uniform* line sources are evaluated. Uniform line sources set $A(l) = 1$ and $\phi(l) = 0$.

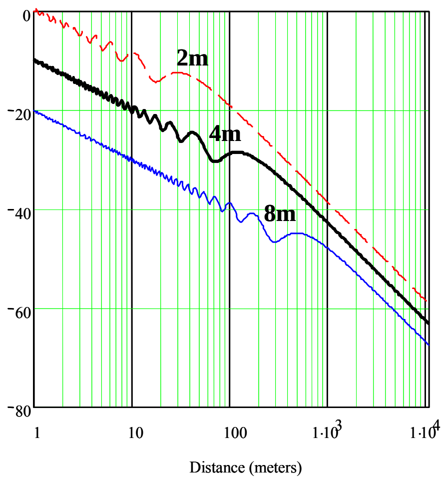

Figure 3: On-axis response (from the midpoint) of a 2, 4, and 8-meter long uniform line source at 8kHz. The 4 and 8-meter response curves are offset by 10dB and 20dB respectively.

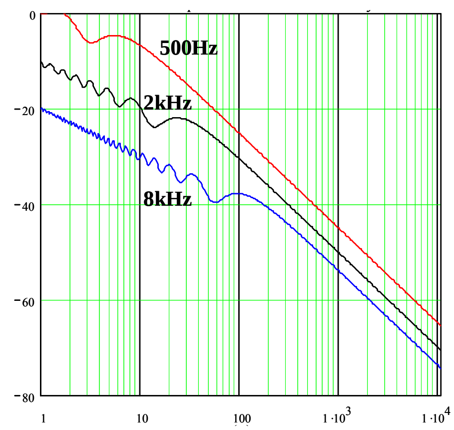

Figure 4: On-axis response (from the midpoint) of a 4-meter uniform line source at 500Hz, 2kHz, and 8kHz. The 2kHz and 8kHz response curves are offset by 10dB and 20dB respectively.

approaches 100 meters. Beyond 100 meters, the pressure amplitude no longer undulates and decreases at 6dB per doubling of distance. The point at which this transition occurs is referred to as the *critical distance*. The region between the source and the critical distance is referred to as the *near field* and the region beyond is the *far field*.

The critical distance varies with source length and frequency. Figure 3 shows the on-axis response of three different length line sources at 8kHz. As length L increases, the critical distance increases. Figure 4 shows the on-axis response of a 4-meter long array at 500Hz, 2kHz, and 8kHz. It shows that the critical distance also increases with frequency.

Figures 2, 3, and 4 summarize the *common wisdom* regarding the on-axis pressure response of line arrays. It holds that the response decreases at 3dB in the near field and at 6dB in the far field. These generalizations, however, significantly understate the complexity of a line source's radiated pressure field. The next sections of this paper describe some of these complexities.

3. MIDPOINT VERSUS ENDPOINT

The conventional approach to determining the pressure response of a line source is to take a path normal to the source beginning at its midpoint. This yields a result, however, that is unique to this path. Choosing the midpoint as a starting point minimizes the apparent "aperture" of the source. If the origin for the path is chosen at a point laterally displaced from the midpoint, the line source appears longer in one direction than the other. This apparent length is maximized at the endpoint of the source. If the pressure is summed along a path normal to the source beginning at the *endpoint* of the source, the critical distance moves further out.

Figure 5 shows a modified geometric construction for calculating the pressure response. The pressure, summed along a path normal to the endpoint of a uniform line source is

$$p_{end}(r) = \int_{-L/2}^{L/2} \frac{e^{-jkr_{end}}}{r_{end}} dl$$

and

$$r_{end} = \sqrt{r^2 + \left(l + \frac{L}{2}\right)^2} .$$

This modified expression for *r'* captures the full aperture of

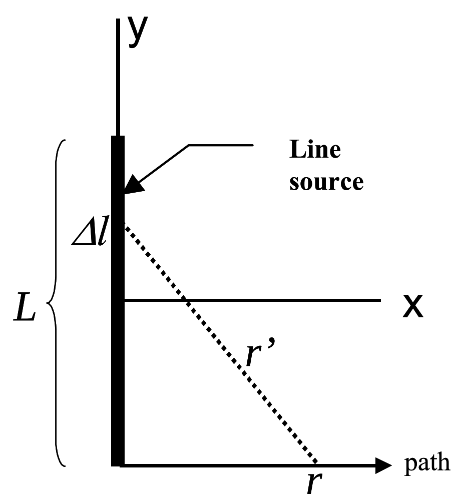

Figure 5: Geometric construction to calculate the pressure response along a path originating at the endpoint.

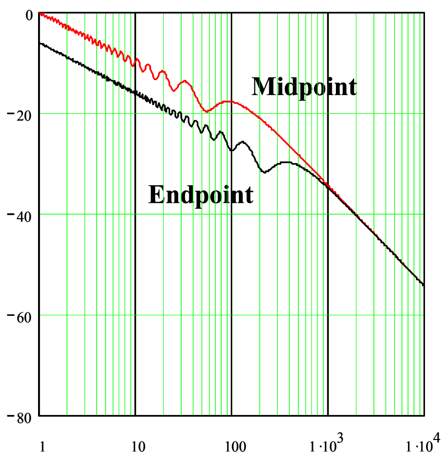

Figure 6: Comparison of midpoint and endpoint pressure response of a 4-m line source at 8kHz.

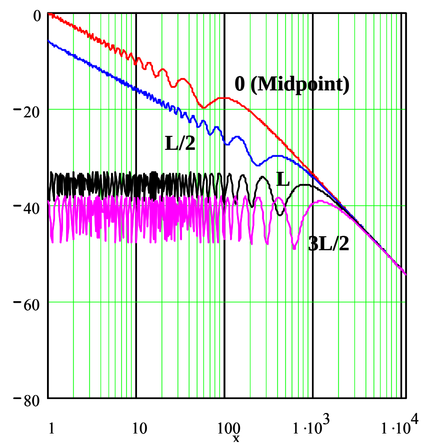

Figure 7: Pressure response along paths normal to the line source at various points of origin. (4-meter, 8kHz)

the line source. Figure 6 compares the on-axis response of a 4-meter long line source at 8khz using the midpoint and endpoint conventions. The last peak of the "midpoint" near field response occurs at approximately 100 meters. The last peak of the "endpoint" response occurs at approximately 400 meters. Note that these response curves are not offset by some arbitrary amount as in Figures 3 and 4. The endpoint response is at a lower level than the midpoint response in the near field. In the far field, the curves converge and yield the monotonically –6dB per doubling of distance fall-off rate described above.

If the origin of the path is moved laterally beyond the endpoint, the distance to the last peak occurs at greater and greater distances. This indicates that the distance to the far field continues to increase as one chooses a path further off the midpoint axis. However, the pressure *levels* fall off quite dramatically once the endpoint is breeched. This is shown in Figure 7. The first two curves labeled "0" and "L/2" are the midpoint and endpoint pressure responses shown in Figure 6. The next two curves are "on-axis" pressure response curves beginning at points one length and three-halves lengths laterally displaced from the midpoint. In this example, the length of the source is 4 meters. The response labeled "L" is the on-axis pressure response, normal to the source, beginning at a point 4 meters from the midpoint of the source. The response labeled "2L/3" starts at 6 meters.

The curves in Figure 7 are "slices" of the pressure field, normal to the source, at increasing lateral distances from the midpoint. They illustrate two important points. First, as the origin of the path is moved laterally off the midpoint, the magnitude of the pressure decreases. It is only –6dB at the endpoint, but approximately –40dB at L and 3L/2. Second, the pressure response curves L and 3L/2 undulate in a fashion that is nearly flat from the source to the far field.

The L and 3L/2 response curves are so low in level relative to the midpoint response that they will have little impact on the overall pressure field. However, the endpoint curve is a material feature of the pressure field and should be considered by sound system designers when using vertical line arrays.

4. PRESSURE FIELD OF A VERTICAL LINE SOURCE

Another way to observe the pressure response of a line source is to compute its entire pressure field. This is obtained by rewriting the expression for the radiated pressure in terms of Cartesian coordinates. The pressure at any point (x,y) is

$$p_{x,y} = \int_{-L/2}^{L/2} \frac{e^{-jkr_{x,y}}}{r_{x,y}} dl$$

where

$$r_{x,y} = \sqrt{x^2 + (y-l)^2}$$

Figure 8 shows the pressure field of a 4-meter line source at 8kHz. The source is located in the upper left corner (not shown) facing to the right. The x-axis in Figure 8[d] is coincident with a path, normal to the source, starting at its midpoint. Only the lower quadrant of the pressure field is shown. The quadrant directly above would be a mirror image symmetrical about the x-axis.

Figure 8 shows the context in which the "slices" of Figure 7 are obtained. The pressure undulations increase in number and decrease in magnitude as the slice moves laterally from the midpoint. Also, the undulations continue out to a greater distance at larger lateral displacement.

[d] The pressure field is plotted on a log-log grid. This is to accommodate the large distances of interest but makes the main lobe of the pressure field appear wider than it would in other formats.

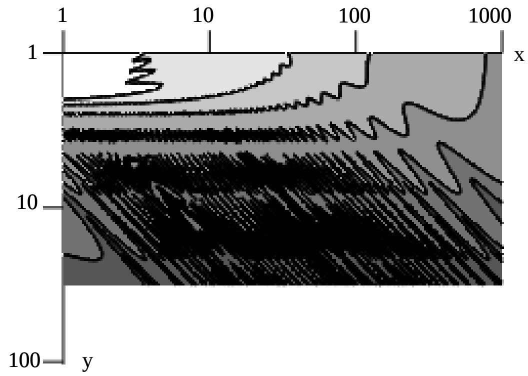

Figure 8: Pressure field of a 4-m line source at 8kHz. The source is located in the upper left corner (not shown) facing to the right. The x-axis in the figure is coincident with a path, normal to the source, starting at its midpoint.

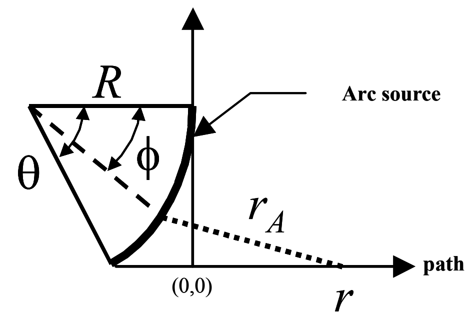

Figure 9: Geometric construction of an arc source.

As we concluded in the previous section, the pressure undulations at large lateral displacements may be immaterial. However, the undulations along the endpoint path are only -6 to -10 dB from the midpoint response and will have a material influence on the sound field. This means, contrary to conventional wisdom, that the pressure in the far field does not monotonically decrease everywhere past the midpoint critical distance.

5. DISTANCE TO THE FAR FIELD - ENDPOINT CONVENTION

Several papers and texts on line arrays offer formulae for estimating the distance to the far field [7,8,9]. Like the on-axis response, these are based on a midpoint convention. The previous section shows that this underestimates the extent of the near field. In this section, we will rewrite the formulae consistent with the endpoint convention.

The distance to the far field of a line source can be estimated by determining at what distance the propagation distance from the ends of a line source are within one-quarter wavelength. That is, referring back to Figure 5, where

$$r = r' - \frac{\lambda}{4}$$

Solving for the critical distance r_c we get

$$r_c = \frac{2L^2}{\lambda} - \frac{\lambda}{8}$$

This shows that the distance to the far field is primarily a function of the square of the source length. Using an example from above, the critical distance for a 4-meter line source at 8kHz is 742 meters. Referring to Figure 6, the endpoint on-axis response curve shows that this is a good estimate of the point where the far field region begins.

As a practical matter, the second term of this expression can be dropped with very little impact on the estimate of the critical distance. Also, it is often useful to express r_c in terms of frequency rather than wavelength. This leaves

$$r_c \approx .006 L^2 f$$

where L is in meters and f is in Hz.

6. ON-AXIS PRESSURE RESPONSE OF A CURVED SOURCE

As mentioned earlier, many line arrays of loudspeakers used in practice are actually curved. This is because pure straight-line arrays of nominal length produce an extremely narrow vertical polar response – often too narrow to reach audiences beneath and slightly in front of the array. A slightly curved array provides superior coverage in this area.

One type of source that provides a wider coverage angle is an *arc* source. It is comprised of radiating elements arranged along a segment of a circle. At all frequencies, it provides a wider directivity response than a straight-line array of the same length. At high frequency, it provides a polar pattern corresponding to the included angle of the arc.

The on-axis pressure response of a curved source can be expressed in a form similar to the earlier expressions for straight-line sources. Figure 9 shows the geometric construction of an arc source with radius R and total included angle □. The arc is facing downward, that is, □ is measured downward from the horizontal. The pressure, using the endpoint convention, of a uniform arc source at distance r is

$$p_A(r) = \int_0^{\theta} \frac{e^{-jkr_A}}{r_A} d\phi$$

where

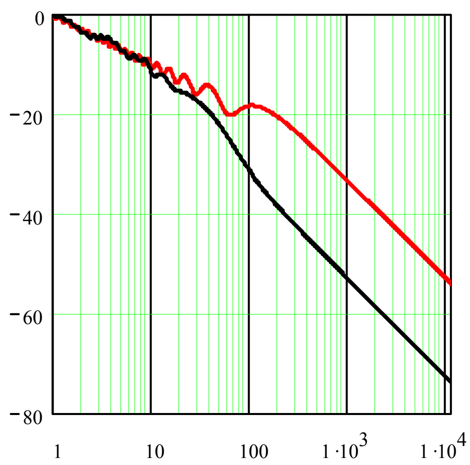

Figure 10: On-axis pressure response of an arc source and a straight line source at 2kHz. The line source has a length of 4 meters. The arc source is 30deg and a radius of 8 meters providing an equivalent total length of 4m.

$$r_A = \left[\left(r + R(1-\cos\phi) \right)^2 + R^2(\sin\theta - \sin\phi) \right]^{\frac{1}{2}}$$

The path is chosen to begin at the bottom of the arc because all radiating elements d□ have clear line-of-sight to any summation point at distance *r*. A path beginning at the top of the arc would produce nonsensical results for large □ and small *r* as *r'* would intersect the source. Also, the origin (0,0) is chosen to be coincident with the leading edge of the arc at the top. This facilitates comparing the on-axis responses of straight line and arc sources.

Figure 10 provides a comparison of the pressure response of equivalent-length straight line and arc sources. Though the arc source is curved only 30 degrees, its on-axis response is smoother in the near field than the one produced by the line source.

Like a line source, the pressure response of an arc source changes with arc length and frequency. Figure 11 shows the on-axis response of three arc sources of various lengths. The different lengths are provided by a constant radius (4m) and different included angles. The critical distance increases with arc length as expected. Figure 12 shows how the pressure response changes with frequency. As with line sources, the critical distance increases with frequency. Note that the transition from the near field to the far field is generally smoother for an arc than for a line source at all lengths and frequency.

7. ON-AXIS RESPONSE OF A J-SOURCE

A J-source [10] a hybrid line source comprised of line source and an curved (arc) source. In sound reinforcement applications, the straight segment is located above the curved segment and is intended to provide the long throw component of the polar response. The curved portion is intended to provide coverage in the area below and in front of the array.

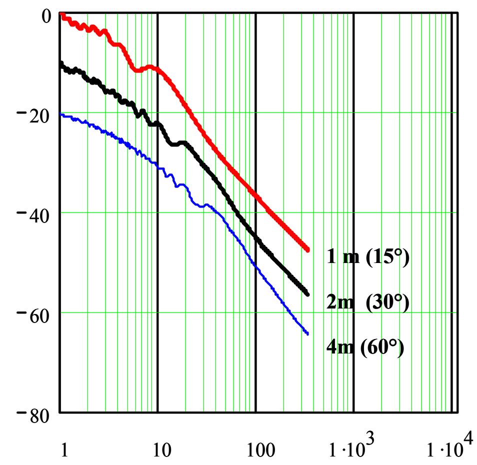

Figure 11: On-axis pressure response of three arc sources (θ = 15°, 30° and 60°) at 8kHz where R=4m. These correspond to arc lengths of 1, 2, and 4 meters.

Together, the segments provide an asymmetric polar response in the vertical plane.

The on-axis pressure response of a J-source is obtained by combining the pressure response functions of the line and curved sources presented above. The geometric construction is shown in Figure 13 where *L* is the length of the straight segment and *R* and □ specify the curved segment. Based on this geometry, the pressure radiated from a uniform J-source is

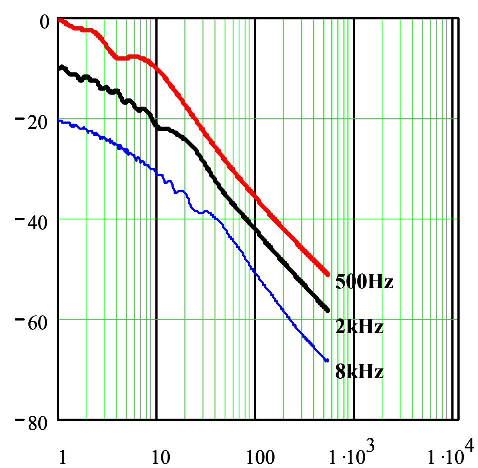

Figure 12: On-axis pressure response of three arc sources at various frequencies where θ = 60° and R= 4m. The 2kHz and 8kHz curves are offset by 10 and 20 dB respectively.

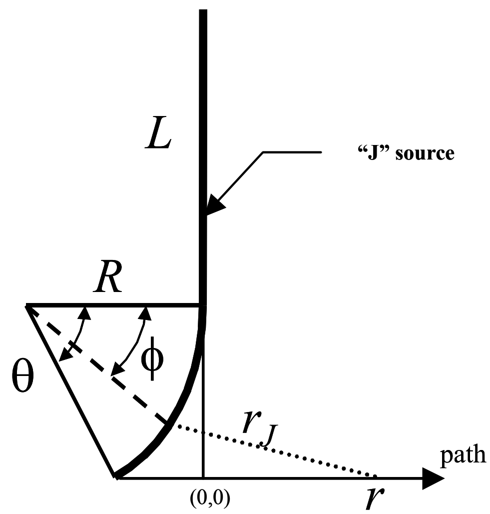

Figure 13: Geometric Construction of the Pressure Field of a J-Source

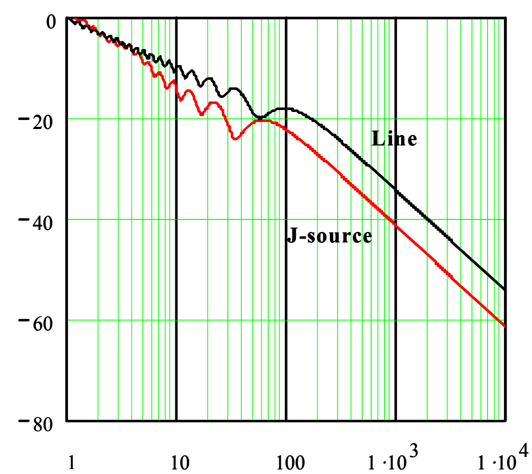

Figure 14: On-axis pressure response of a J-source where L=2m, R=2m, θ=60° and f=2kHz.

$$p_J(r)=\int_0^L \frac{e^{-jkr_L}}{r_L}dl+R\int_0^\theta \frac{e^{-jkr_A}}{r_A}d\phi$$

where

$$r_L=\sqrt{r^2+(R\sin\theta+l)^2}$$

and

$$r_A=\sqrt{(r+R(1-\cos\phi))^2+R^2(\sin\theta-\sin\phi)^2}$$

Figure 14 shows the on-axis pressure response of equivalent-length straight line and J-sources. The straight segment of the J-source dominates the response, producing undulations in the near field very similar to those of the straight line source. However, the on-axis aperture of the J-source is foreshortened relative to the equal-length line source so the distance to the far field is marginally shorter.

8. ON-AXIS RESPONSE OF A SPIRAL SOURCE

Like the J-source, a spiral source [11] provides an asymmetric polar response in the vertical plane. However, unlike the J-source, it is a continuous curve rather than two distinct segments. The curvature increases with distance along the curve. This results in an upper portion that is largely - but not perfectly - straight, and a lower portion that is curved downward. Spiral sources provide polar response curves that are remarkably constant with frequency.

A spiral source is fully defined by its length *L* and terminal angle □. The pressure response is determined by summing the radiation from each elemental length □*L* along the source. *ΔL* is chosen to be a small fraction of the shortest wavelength of interest[e]. The total number of elemental lengths *M* is

$$M=\frac{L}{\Delta L}$$

and the incremental angle between the elements is

$$\Delta\psi=\frac{2\Omega}{M(M+1)}.$$

The spiral can then be expressed in parametric form as

$$x(s)=\sum_{\eta=0}^{s}-\sin\left[\tfrac{1}{2}\eta(\eta+1)\Delta\psi\right]\Delta L$$

and

$$y(s)=\Delta L+\sum_{\eta=0}^{s}-\cos\left[\tfrac{1}{2}\eta(\eta+1)\Delta\psi\right]\Delta L.$$

The cross-section of a spiral source is shown is Figure 15. Its on-axis pressure response is

$$p_S(r)=\sum_{\sigma=0}^{M}\frac{e^{-jkr_S}}{r_S}$$

[e] For instance, to obtain the directivity function up to 4kHz, □*L* should be less than ¼ wavelength, i.e. approximately 0.02 meters.

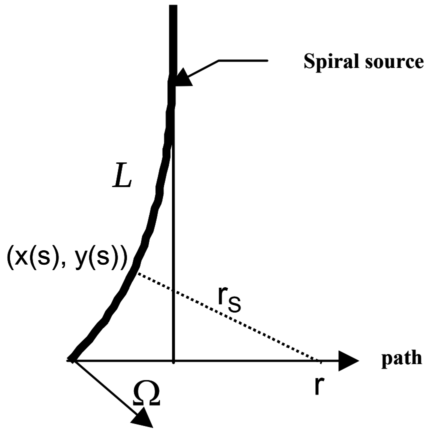

Figure 15: Geometric construction of a spiral source,

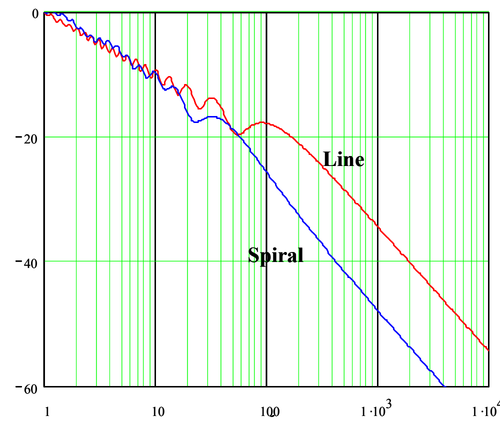

Figure 16: Pressure response comparison of a 45° terminal angle, 4 meter long spiral source and line source of the same length at 2kHz.

where

$$r_S = \sqrt{(r - x(\sigma))^2 + (y(M) - y(\sigma))^2} \ .$$

Figure 16 compares the on-axis response curves of equivalent-length straight-line and spiral sources. These curves show that the spiral source has reduced undulations in the near field and a smoother transition from the near field to the far field. Coupled with its favorable polar response characteristics, a spiral source provides an excellent geometry upon which to base loudspeaker arrays for sound reinforcement applications.

9. SUMMARY

This paper shows that the conventional method for calculating the critical distance of line sources understates the extent of the undulations in the near field. The conventional method takes a path normal to the source beginning at its midpoint and provides a limited picture of the radiated pressure field. Choosing the endpoint as the origin of the path takes the full aperture of the line source into account and shows that the pressure undulations characteristic of the near field continue well past the midpoint-derived critical distance. This result is important to sound system designers because they will not achieve a monotonically decreasing pressure response everywhere past the midpoint critical distance.

The endpoint convention is applied to the analysis of hybrid line sources types including arc, J, and spiral. These generally provide a smoother transition from the near field to the far field than straight line sources.

10. REFERENCES

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