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"J" and "Spiral" Line Arrays

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ABSTRACT

Straight-line arrays produce highly directional polar response curves in the vertical plane resulting in high on-axis gain. In many venues, however, it is useful to blend this high on-axis gain with improved response in the near field beneath and in front of the array. To accomplish this the lower section of a straight-line array is curved. This paper derives the directivity functions of two such arrays, namely the "J-Array" and the "Spiral Array."

INTRODUCTION

Line arrays of loudspeakers are used by sound system designers to obtain a narrow directivity response in the vertical plane. For long arrays at high frequency it is not uncommon to produce extremely narrow angles - in some cases, angles of just a fraction of a degree are obtained [1,2]. This can be useful in certain venues requiring a very long throw but limits a line array's applicability where broader coverage is required. In particular, it is often useful to have an asymmetric coverage pattern in the vertical plane where high gain is used to reach seats in the rear of a venue and a broader pattern is used to fill-in the seats in the front.

A common approach to obtain such an asymmetric pattern is to curve the lower section of a line array. In this case the top elements of the array are arranged in a straight line while the lower elements are arranged to point progressively downward. When analyzing the directional characteristics of such an array it is important to sum the contributions of all of the elements and not treat the elements separately. Except in cases where the directional response of an individual element is highly directional, each element will have some contribution to the overall response of the array.

Contemporary texts [3,4] provide directivity functions for straight-line and curved sources[a] but none treat combinations of these or other shapes. This paper provides solutions for two general classes of asymmetric arrays - the "J-Array" and the "Spiral Array." A J-array is a combination of a straight and a curved source. A spiral array has a continuous, progressively curved profile.

The paper begins with a derivation of the directivity function of a straight-line source. This derivation can be found in several texts and is restated here for the convenience of the reader. It provides the method by which we will derive the directivity functions of the J and spiral arrays. These arrays have many different embodiments and their directional characteristics change with dimension, amplitude shading and frequency. Polar response curves are shown for a few representative examples. The J-array provides useful asymmetric

[a] *Arrays* and *sources* are used here interchangeably. Line *arrays* generally refer to an array of acoustical radiators such as loudspeakers arranged in a line. A line *source* is a mathematical representation of a large number of infinitely small radiating segments of a line.

polar patterns and the spiral array provides an asymmetric polar response remarkably constant with frequency.

LINE ARRAY

A line source can be modeled as a continuum of infinitely small line segments distributed along a line [5]. The acoustic pressure radiated is

$$pressure = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{A(x)}{r(x)} e^{-j(kr(x)+\phi(x))} dx$$

where **L** is the length of the line source, *A(x)* is the amplitude function along the line, *k* is the wave number, $\phi(x)$ is the phase function along the line, and *r(x)* is the distance from any segment along the line to the point of observation **P**.

The evaluation of this expression is greatly simplified if we assume that the observation point **P** is a large distance away. That is, that the distance is much greater than the length of the array and that the distance to **P** from any two segments along the line are approximately equal. This allows us to bring the *1/r(x)* term in front of the integral since

$$\frac{1}{r(x)} \approx \frac{1}{r(\frac{L}{2})} \approx \frac{1}{r(\frac{-L}{2})} \approx \frac{1}{r}.$$

Conversely, the *r(x)* term in the exponential must be dealt with. This is because the relatively small distance differences to **P** from any two segments are not small compared to a wavelength. Figure 1 shows that *r(x)* term in the exponent, which we refer to as the *relative distance function*, can be expressed as

$$r(x) = x\sin(\alpha)$$

where α is the angle between a line normal to the axis of the source and a line from the source to **P**.

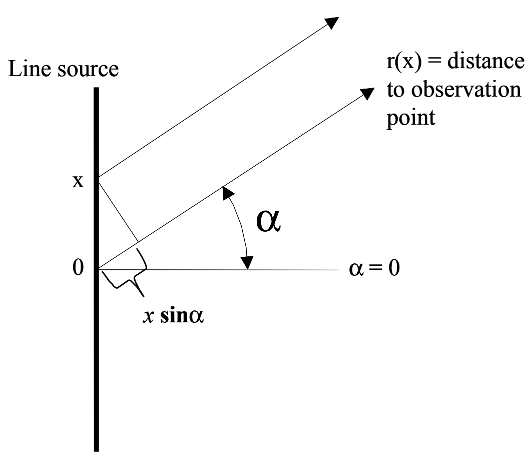

Figure 1: Geometric construction of a straight-line source.

Substituting for the relative distance term *r(x)*, the far field pressure at angle α of a continuous line source is

$$pressure(\alpha) = \frac{1}{r} \int_{-\frac{L}{2}}^{\frac{L}{2}} A(x) e^{-j(kx\sin\alpha+\phi(x))} dx$$

The *directivity function R(α)* of a line source is defined as the magnitude of the pressure at angle α over the magnitude of the maximum pressure that can be obtained. That is,

$$R_{Line}(\alpha) = \frac{|pressure|}{|pressure_{\max}|}.$$

The maximum radiated pressure is obtained when all segments along the line radiate in phase, i.e. the exponential function equals unity[b]. The maximum pressure is given as:

$$pressure_{\max} = \frac{1}{r} \int_{-\frac{L}{2}}^{\frac{L}{2}} A(x) dx.$$

The general form of the directivity function *R(α)* of a line source is then

$$R_{Line}(\alpha) = \frac{\left| \int_{-\frac{L}{2}}^{\frac{L}{2}} A(x) e^{-j(kx\sin\alpha+\phi(x))} dx \right|}{\left| \int_{-\frac{L}{2}}^{\frac{L}{2}} A(x) dx \right|}.$$

For a line source with uniform amplitude and phase, i.e. *A(x)* =1 and $\phi(x)$ = 0, the directivity function becomes[c]

$$R_{Line}(\alpha) = \frac{1}{L} \left| \int_{-\frac{L}{2}}^{\frac{L}{2}} e^{-jkx\sin\alpha} dx \right|.$$

[b] Note that the maximum pressure at any given distance and frequency may never actually be obtained.

[c] Solving the integral puts the directivity function in a form commonly found in contemporary texts. It is generally expressed as

$$R_{Line}(\alpha) = \left| \frac{\sin(k\frac{L}{2}\sin\alpha)}{k\frac{L}{2}\sin\alpha} \right|.$$

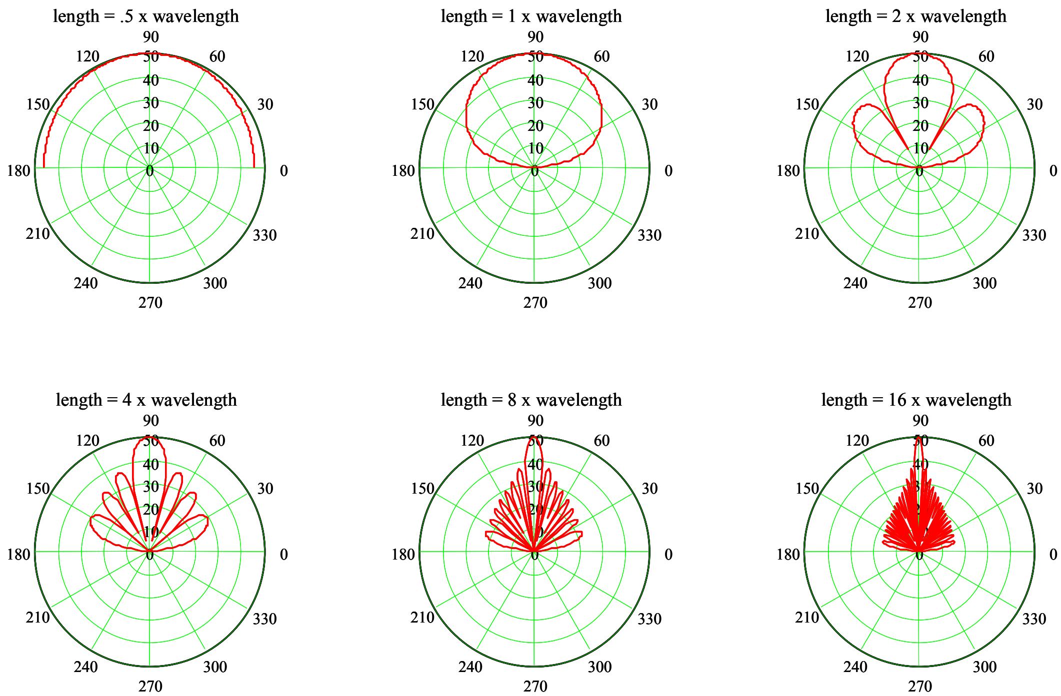

Figure 2: Polar response curves of a uniform line source.

Figure 2 shows the polar response curves[d] of a uniform amplitude and phase line array at various ratios of array length and wavelength. The polar response is wide at low ratios of L/λ. As the ratio increases the directivity pattern narrows and exhibits lobes and nulls.

CURVED ARRAY

A curved array is comprised of radiating elements arranged along a segment of a circle. It generally provides a wider directivity response than a straight-line array. At high frequency, it provides a polar pattern corresponding to the included angle of the arc.

The derivation of the directivity function of a curved source follows the same steps described above for the line source. Figure 3 shows the geometric construction of a curved source with radius **R** and total included angle θ.

As before, the far field directivity function is driven by the relatively small distance difference to the far field from any two points along the curve. The relative distance function for a curved source $r_C(\sigma)$ is

$$r_C(\sigma) = 2\mathrm{R}\sin\left[\frac{\sigma}{2}\right]\sin\left[\frac{\sigma}{2}+\alpha\right].$$

The far field directivity function of a curved source can then be expressed as

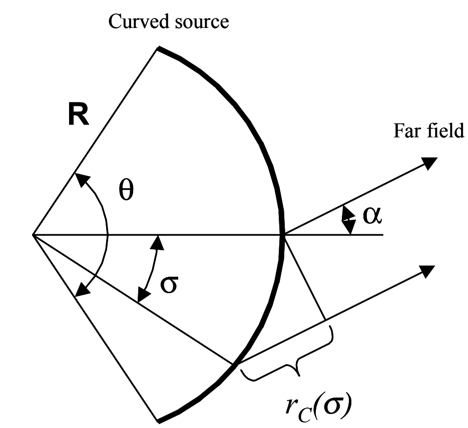

Figure 3: Geometric construction of a curved source.

[d] A polar response curve is the directivity function expressed in decibels and plotted on a polar chart. The on-axis pressure is used as the reference pressure, i.e.

$$\text{Polar response} = 20\log\frac{R(\alpha)}{R(0)}.$$

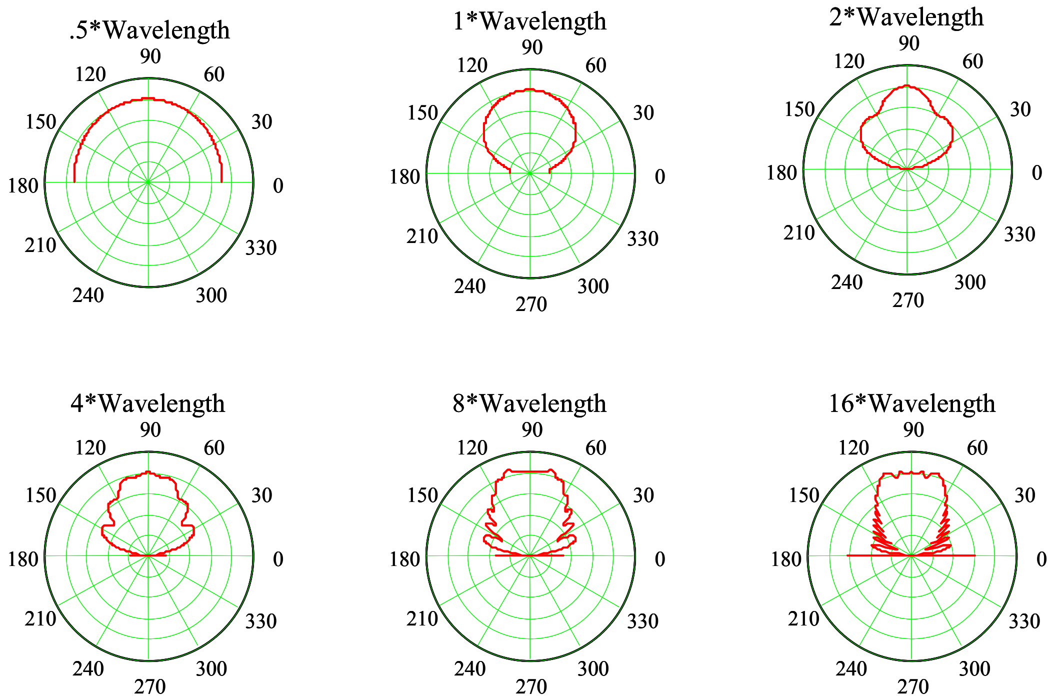

Figure 4: Polar response curves of a 60 degree curved source.

$$R_{Curved}(\alpha) = \left| \frac{\int_{\frac{-\theta}{2}}^{\frac{\theta}{2}} A(\sigma) e^{-j[kr_c(\sigma)+\phi(\sigma)]} d\sigma}{\int_{\frac{-\theta}{2}}^{\frac{\theta}{2}} A(\sigma) d\sigma} \right| .$$

If we assume uniform amplitude and phase along the curved array, i.e. $A(\sigma) = 1$ and $\phi(\sigma) = 0$, the directivity function is[e]:

$$R_{Curved}(\alpha) = \frac{1}{\theta} \left| \int_{\frac{-\theta}{2}}^{\frac{\theta}{2}} e^{-jkr_c(\sigma)} d\sigma \right| .$$

The polar response curves of a 60° curved source at various ratios of radius and wavelength are shown in Figure 4. Generally, these are wide for low ratios of **R**/λ and obtain the included angle of the arc at higher ratios.

"J" ARRAYS

A J-Array is comprised of a line array and a curved array. Generally the straight segment is located above the curved segment and is intended to provide the long throw component of the polar response. The curved segment is intended to provide coverage in the relative near field[f] below and in front of the array. Together, the segments provide an asymmetric polar response in the vertical plane.

The directivity function of a J-array is obtained by combining the directivity functions of the line and curved arrays presented above. The geometric construction is shown in Figure 5 where **L** is the length of the straight segment and **R** and θ specify the curved segment. We assume that the straight and curved segments are

[e] This integral does not have a convenient closed form solution similar to the one obtained for the line array. Wolff and Malter [5] provide a point source summation version of the directivity function as follows:

$$R_{Curved}(\alpha) = \frac{1}{2m+1} \left| \sum_{n=-m}^{n=m} \cos\left[\frac{2\pi R}{\lambda}\cos(\alpha + n\phi)\right] + i \sum_{n=-m}^{n=m} \sin\left[\frac{2\pi R}{\lambda}\cos(\alpha + n\phi)\right] \right|$$

where m is an integer, 2m+1 is the number of point sources, and ϕ is the angle subtended between any two adjacent point sources.

[f] The term "relative near field" indicates that in this context the "near field" is not an explicit reference to the Fresnel zone [6] but rather a qualitative measure of being relatively close to the array as opposed to being far away.

adjacent and that the center point of the arc is on a line perpendicular to the straight segment and through its lower endpoint.

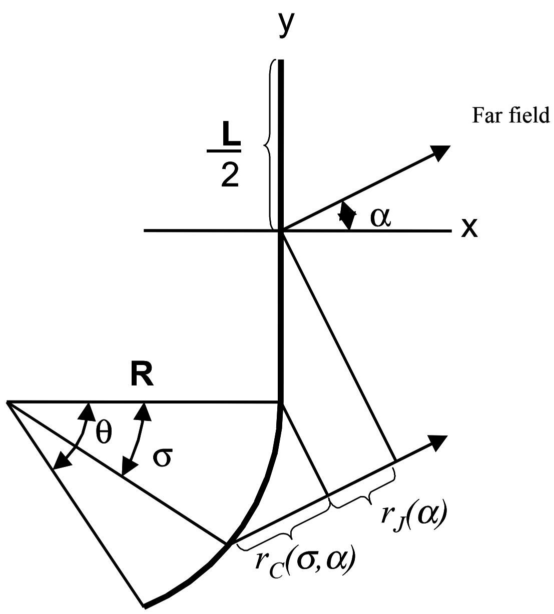

Figure 5: Geometric construction of a J-array.

If we choose the center point of the line section as the origin, then the pressure radiated from the line is

$$R_{Line}(\alpha) = \left| \frac{1}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} e^{-jkr_L(x)} dx \right|$$

where

$$r_L(x) = x\sin(\alpha)$$

as before. The curved segment is rotated by θ/2 relative to the horizontal so the limits of integration must be changed from before. The directivity function of the rotated curved array is

$$R_{Curved}(\alpha) = \left| \frac{1}{\theta} \int_0^{\theta} e^{-jkr_C(\sigma)} d\sigma \right|.$$

where

$$r_C(\sigma) = 2R\sin\left[\frac{\sigma}{2}\right]\sin\left[\frac{\sigma}{2}+\alpha\right]$$

as before.

To properly sum the radiated pressure from the line and curve segments, a new function is required to express the relative distance difference between them. Referring to Figure 5, it is given as

$$r_J(\sigma) = \frac{L}{2}\sin(\alpha).$$

The directivity function of a J-array is then

$$R_J(\alpha) = \frac{1}{A_L L + A_C R\theta} \left| A_L \int_{-\frac{L}{2}}^{\frac{L}{2}} e^{-jkr_L(\alpha)} + A_C R \int_0^{\theta} e^{-jk[r_C(\sigma)+r_J(\sigma)]} d\sigma \right|$$

where A_L and A_C are the amplitudes-per-unit-length of the straight and curved segments. If we assume that the amplitudes-per-unit-length are uniform over the line and curve segments, the relative source strengths are proportional to their relative lengths. The contributions of the line segment and the curved segment to the polar response of a uniform J-array are shown in Figure 6.

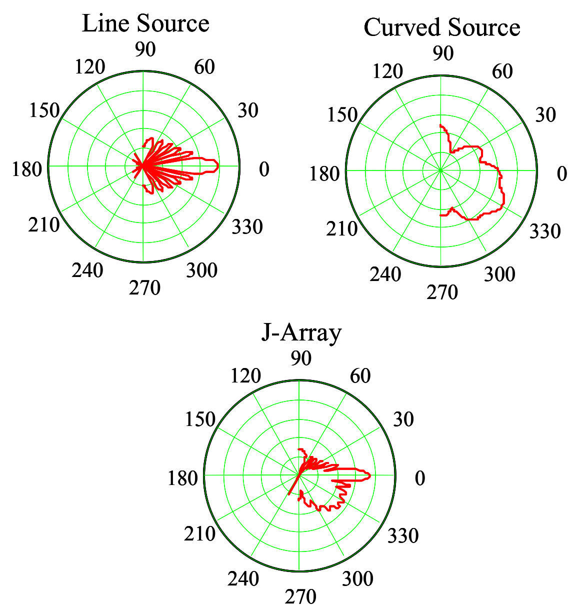

Figure 6: Polar response curves at 1kHz of a 2-meter long line array, a 60 degree curved source with radius of 1 meter, and the corresponding J array.

As expected, the line segment provides long throw and the curved segment provides a relatively wide angle of coverage rotated downward. The response of the J-array is a blend of the two.

The directivity of a J-array changes with the length of the line segment, the radius and angle of the curved segment, the relative amplitude of the two segments, and frequency. Figure 7 shows the directivity response of a J-array with a two-meter long line segment, a one-meter radius, a 60° included angle and equal amplitudes-per-unit-length. The polars show that the straight segment of the J-Array dominates the response and produces a very narrow beam of energy,

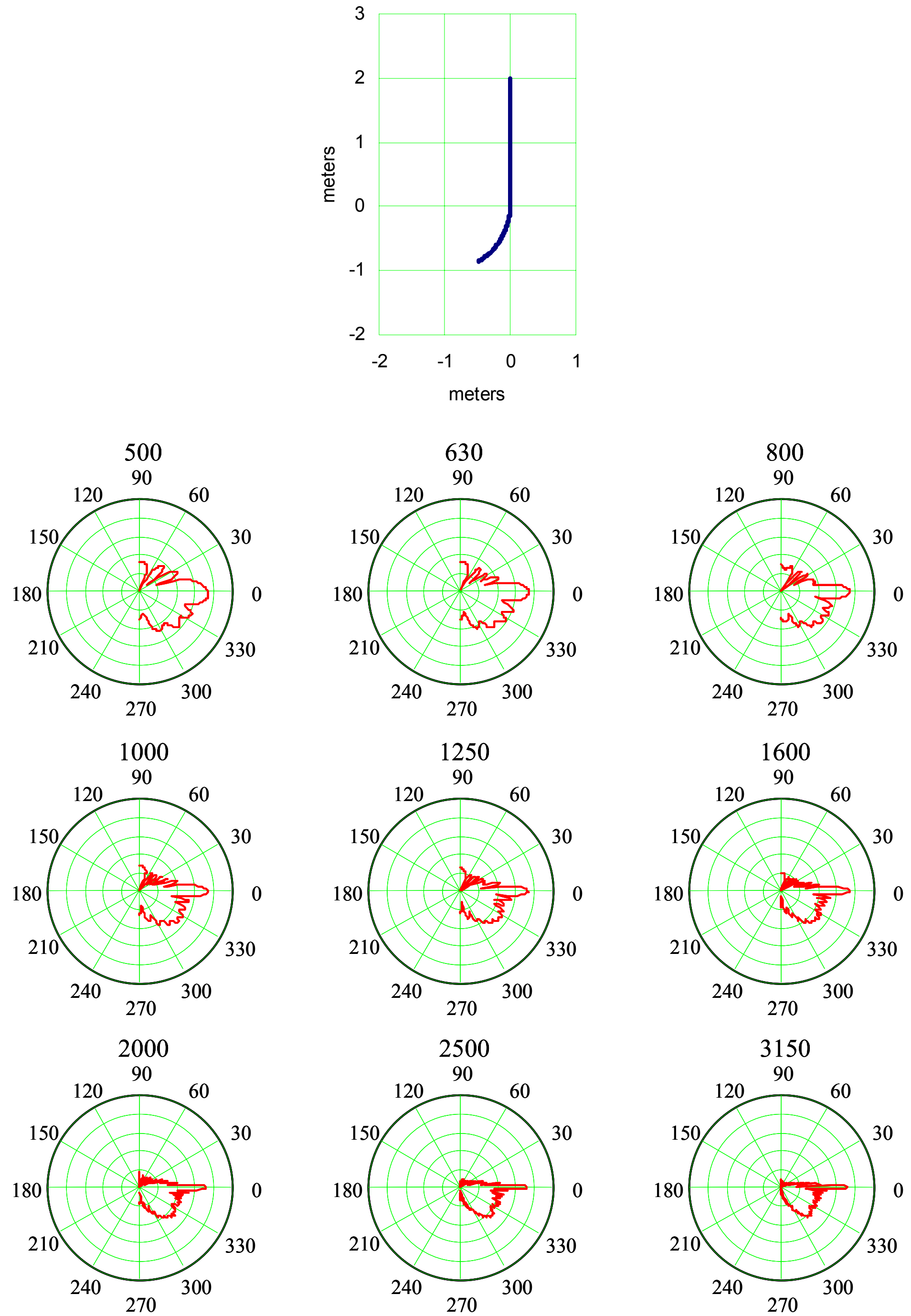

Figure 7: Polar response curves of J-Array where L = 2 meters, R = 1 meter, θ = 60 degrees, and $A_L = A_C = 1$.

particularly at high frequency. The curved segment does not fully balance the high gain of the straight segment.

There are several approaches to providing a more balanced response. One is to make the straight segment shorter thereby reducing the gain. A second is to increase A_C relative to A_L. This is demonstrated in Figure 8. This J-array has a one-meter straight segment (as

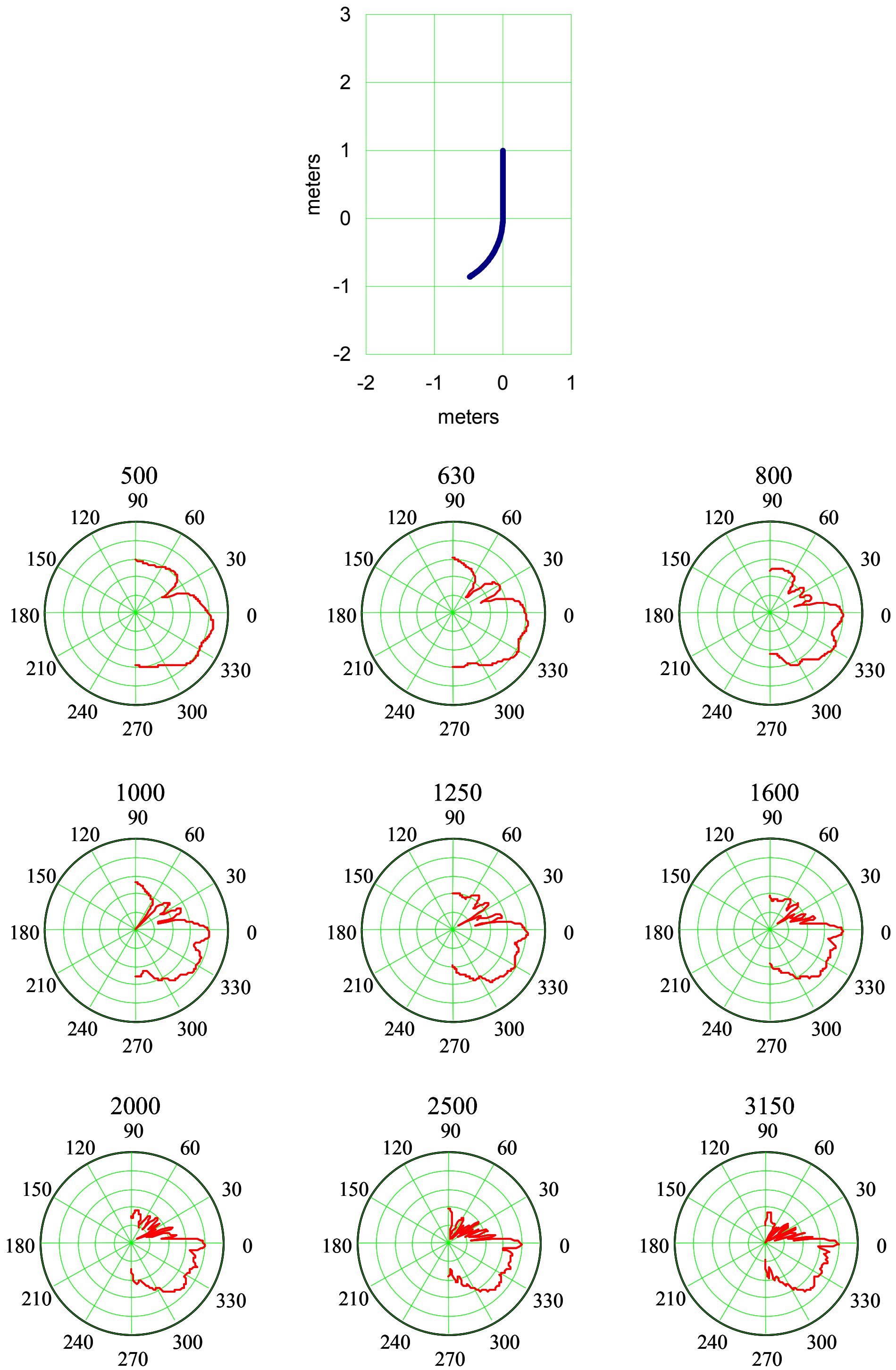

Figure 8: Polar response curves of J-Array where L = 1 meter, R = 1 meter, θ = 60 degrees, A_L = 1, and A_C = 2.

opposed to 2-meter in the previous example) and $A_C = 2A_L$ (instead of $A_C=A_L$). The polar response of this J-array is considerably more balanced than the uniform J-array.

Another approach to provide a more balanced response is to begin the curvature at the top of the array and to not allow any portion of the array to be purely straight. An array that provides this profile is the Spiral Array, discussed in the next section.

SPIRAL ARRAY

Like the J-array, a spiral array provides an asymmetric polar response in the vertical plane. However, unlike the J-array, it is a continuous curve rather than two distinct segments. The curvature increases with distance along the curve. This results in an upper portion that is largely - but not perfectly - straight, and a lower portion that is curved downward.

There are many types of spirals providing various rates of curvature. The relevant set of spirals for arrays of loudspeaker enclosures are those for which the curvature changes at predetermined intervals of length along the spiral. This length interval corresponds to the height of the enclosure.

An *arithmetic spiral* is one for which the angle between successive enclosures changes by a predetermined angle $\Delta\theta$. For example, the top box might be hung at 0°, the next box at 1°, the next at 2° and so on. This defines a spiral where the angle of the n^{th} box is oriented to the vertical axis by 0°, 1°, 3°, 6°, 10° and so on - an arithmetic expansion. An incremental angle of 2° would yield 0°, 2°, 6°, 12°, 20° and so on. The terminal angle can be given as

$$\Omega = \frac{1}{2} n(1+n)\Delta\theta$$

where n is the number of boxes. The total length of the array is

$$L = nH$$

where H is the height of a single box. These two terms, Ω and L, fully define an arithmetic spiral.

The directivity function of an arithmetic spiral array is derived in the same manner used earlier for the line, curved, and J arrays. The pressure radiated along the array is summed at a point in the far field. The shape of the polar response curves will be determined primarily by the relative distance function.

The first step is to express the spiral as a continuum of small radiating segments of length ΔL. ΔL should be chosen to be a small fraction of the shortest wavelength of interest[g]. The total number of segments is then

$$m = \frac{L}{\Delta L}$$

and the incremental angle between the elements is

$$\Delta\psi = \frac{2\Omega}{m(m+1)}.$$

The spiral can then be expressed in parametric form as

$$x(s) = \sum_{\eta=0}^{s} -\sin\left[\tfrac{1}{2}\eta(\eta+1)\Delta\psi\right]\Delta L$$

and

[g] For instance, to obtain the directivity function up to 4kHz, DL should be less than ¼ wavelength, i.e. approximately 0.02 meters.

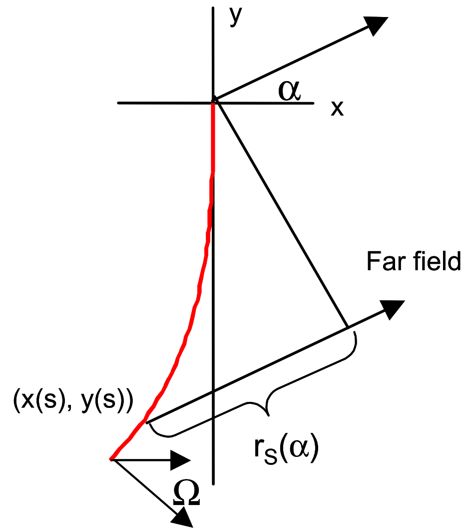

Figure 9: Geometric construction of an arithmetic spiral array.

$$y(s) = \Delta L + \sum_{\eta=0}^{s} -\cos\left[\tfrac{1}{2}\eta(\eta+1)\Delta\psi\right]\Delta L.$$

The geometric construction of an arithmetic spiral is shown in Figure 9. The relative distance function r_S is

$$r_S(s,\alpha) = \sin\left[\alpha - \tan^{-1}\left(\frac{x(s)}{y(s)}\right)\right]\sqrt{x(s)^2 + y(s)^2}$$

where s is an index along the spiral. The directivity function of an arithmetic spiral array is then

$$R_{Spiral}(\alpha) = \frac{1}{m+1}\left|\sum_{s=0}^{m} e^{-jkr_S(s,\alpha)}\right|.$$

The polar response of an arithmetic spiral array is remarkably constant with frequency. Figure 10 shows the cross-section of an arithmetic spiral array comprised of ten loudspeaker enclosures.

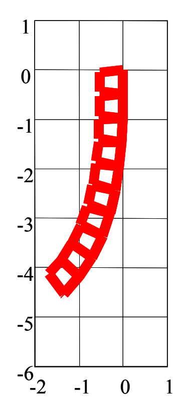

Figure 10: Arithmetic spiral array 5 meters long with a terminal angle of 45 degrees.

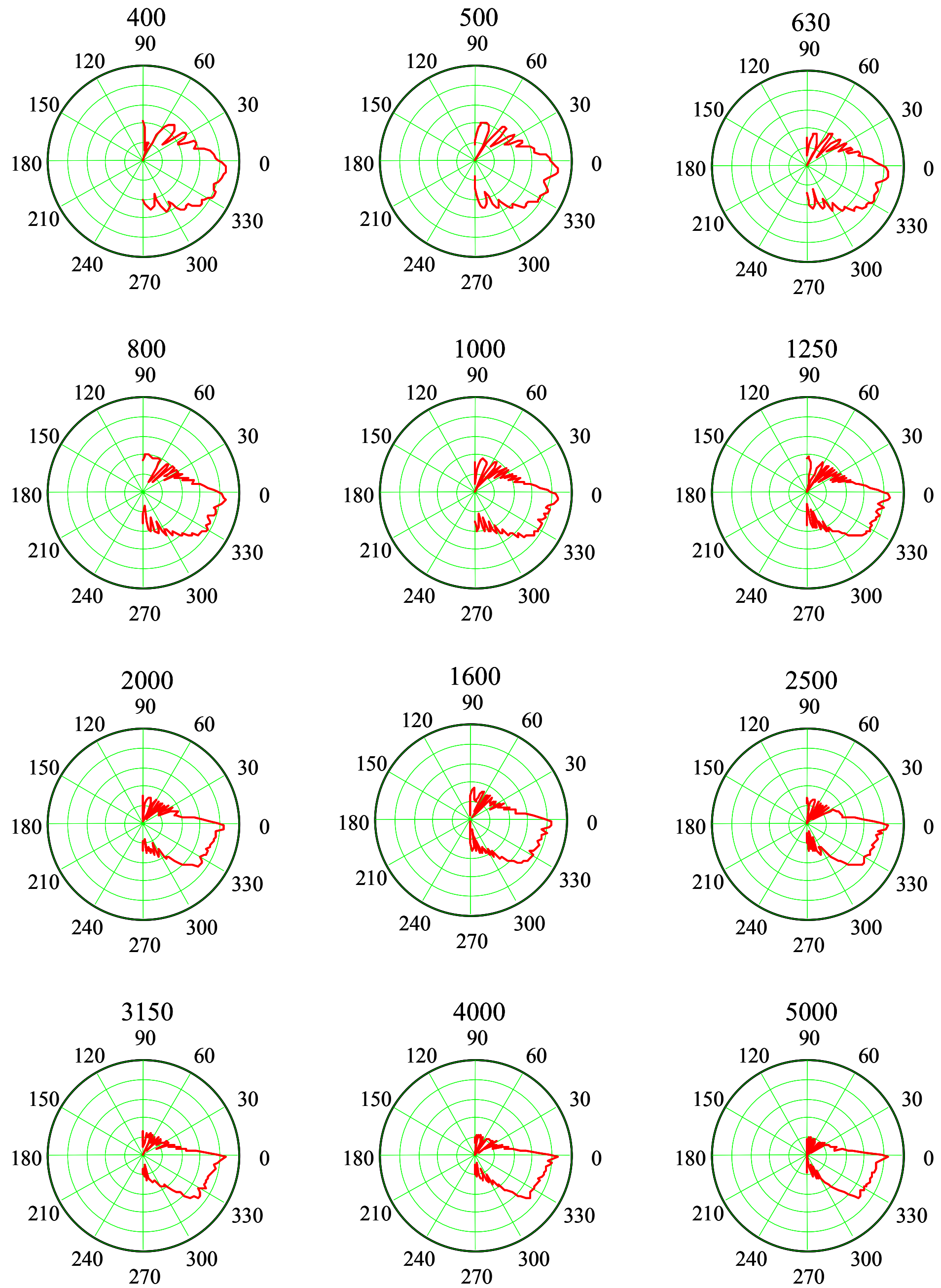

Figure 11: Polar response curves of arithmetic spiral array 5 meters long with a terminal angle of 45 degrees.

Each enclosure is 0.5 meter high giving the array a total length of 5 meters. The terminal angle is 45°.

The polar response curves from 400 Hz to 5kHz are shown in Figure 10. The response is very constant with frequency and exhibits minimal side-lobe structure.

SUMMARY

This paper derives the directivity function of two new array types, the J-array and the arithmetic spiral array. Both of these array types provides a blend of long throw and near-field coverage useful in many sound reinforcement venues. Polar response curves of two J-arrays are shown to illustrate how the directional characteristics change with dimension, amplitude shading and frequency. A set of polar response curves of an arithmetic spiral array illustrates how constant with frequency its polar response can be.

Our analysis of hybrid line arrays continues. Areas of interest include the on-axis pressure response of J-arrays, spiral arrays, and others. Also, there are several types of spiral arrays that may prove useful for professional sound reinforcement applications. For instance, a geometric rate of curvature may have advantages over the arithmetic spiral presented in this paper. These will likely be topics of future papers.

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$$\theta_{-6dB} \approx \frac{24{,}000}{fl} \ .$$

This shows that a 4-meter long line array at 10kHz would have a quarter-power angle of just 0.6 degrees.

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